Package 'WRI'

November 23, 2020

Type Package

Title Wasserstein Regression Inference

Version 0.1.0

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Description An implementation of the methodologies described in Alexander Petersen, Xi Liu and Afshin A. Divani (2019) <arXiv:1910.13418>, including global F tests, partial F tests, intrinsic Wasserstein-infinity bands and Wasserstein density bands.

License GPL-2

Encoding UTF-8

LazyData true

LinkingTo Rcpp, RcppArmadillo

Depends R (>= 3.6.0)

Imports fdapace (>= 0.2.0), fdadensity (>= 0.1.2), Rfast (>= 1.9.8), CVXR (>= 0.99.7), expm (>= 0.999-4), ggplot2 (>= 3.2.1), gridExtra (>= 2.3), stats, Rcpp (>= 1.0.3), locfit (>= 1.5-9.1), mvtnorm (>= 1.1-0), locpol (>= 0.7), modeest (>= 2.4.0), methods, rlang

RoxygenNote 7.0.2

Suggests knitr, rmarkdown, testthat (>= 2.1.0)

VignetteBuilder knitr

NeedsCompilation yes

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Repository CRAN

Date/Publication 2020-11-23 08:00:06 UTC

R topics documented:

`confidenceBands` *Confidence Bands for Wasserstein Regression*

Description

Confidence Bands for Wasserstein Regression

Usage

```
confidenceBands(
  wass_regress_res,
  Xpred_df,
  level = 0.95,
  delta = 0.01,
  type = "density",
  figure = TRUE,
  fig_num = NULL
)
```

Arguments

`wass_regress_res`	an object returned by the `wass_regress` function
`Xpred_df`	k-by-p matrix (or dataframe, or named vector) used for prediction. Note that Xpred_df should have the same column names with Xfit_df used in wass_regress_res
`level`	confidence level
`delta`	boundary control value in density band computation. Must be a value in the interval (0, 1/2) (default: 0.01)
`type`	'density', 'quantile' or 'both'

- 'density': density function bands will be returned (and plotted if `figure = TRUE`)

- 'quantile': quantile function and CDF bands will be returned (and plotted if `figure = TRUE`)
- 'both': three kinds of bands, density function, quantile function and CDF bands will be returned (and plotted if `figure = TRUE`)

`figure` logical; if TRUE, return a sampled plot (default: TRUE)

`fig_num` the fig_num-th row of `Xpred_df` will be used for visualization of confidence bands. If NULL, then `fig_num` is randomly chosen (default: NULL)

Details

This function computes intrinsic confidence bands for `Xpred_df` if `type = 'quantile'` and density bands if `type = 'density'`, and visualizes the confidence and/or density bands when `figure = TRUE`.

Value

a list containing the following lists:

`den_list:`
- fpred: k-by-m matrix, predicted density function at Xpred_df.
- f_ux: k-by-m matrix, upper bound of confidence bands of density functions.
- f_lx: k-by-m matrix, lower bound of confidence bands of density functions.
- Qpred: k-by-m matrix, f_lx[i,], f_ux[i,] and fpred[i,] evaluated on Qpred[i,] vector.

`quan_list:`
- Qpred: k-by-m matrix of predicted quantile functions.
- Q_ux: k-by-m matrix of upper bound of quantile functions.
- Q_lx: k-by-m matrix of lower bound of quantile functions.
- t_vec: a length m vector - common grid for all quantile functions.

`cdf_list:`
- fpred: k-by-m matrix, predicted density function.
- Fpred: k-by-m matrix, predicted cumulative distribution functions.
- F_ux: k-by-m matrix, upper bound of cumulative distribution functions.
- F_lx: k-by-m matrix, lower bound of cumulative distribution functions.
- Fsup: k-by-m matrix, fpred[i,], F_lx[i,], F_ux[i,] and Fpred[i,] evaluated on Fsup[i,] vector.

Examples

```
alpha = 2
beta = 1
n = 50
x1 = runif(n)
t_vec = unique(c(seq(0, 0.05, 0.001), seq(0.05, 0.95, 0.05), seq(0.95, 1, 0.001)))
set.seed(1)
quan_obs = simulate_quantile_curves(x1, alpha, beta, t_vec)
Xfit_df = data.frame(x1 = x1)
res = wass_regress(rightside_formula = ~., Xfit_df = Xfit_df,
                   Ytype = 'quantile', Ymat = quan_obs, Sup = t_vec)
confidence_Band = confidenceBands(res, Xpred_df = data.frame(x1 = c(-0.5,0.5)),
type = 'both', fig_num = 2)
```

```
data(strokeCTdensity)
predictor = strokeCTdensity$predictors
dSup = strokeCTdensity$densitySupport
densityCurves = strokeCTdensity$densityCurve
xpred = predictor[2:3, ]

res = wass_regress(rightside_formula = ~., Xfit_df = predictor,
Ytype = 'density', Ymat = densityCurves, Sup = dSup)
confidence_Band = confidenceBands(res, Xpred_df = xpred, type = 'density', fig_num = 1)
```

---

den2Q_qd *convert density function to quantile and quantile density function*

Description

convert density function to quantile and quantile density function

Usage

```
den2Q_qd(densityCurves, dSup, t_vec)
```

Arguments

densityCurves	n-by-m matrix of density curves
dSup	length m vector contains the common support grid of the density curves
t_vec	common grid for quantile functions

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globalFtest *global F test for Wasserstein regression*

Description

global F test for Wasserstein regression

Usage

```
globalFtest(
  wass_regress_res,
  alpha = 0.05,
  permutation = FALSE,
  numPermu = 200,
  bootstrap = FALSE,
  numBoot = 200
)
```

Arguments

wass_regress_res	an object returned by the wass_regress function
alpha	type one error rate
permutation	logical; perform permutation global F test (default: FALSE)
numPermu	number of permutation samples if permutation = TRUE
bootstrap	logical; bootstrap global F test (default: FALSE)
numBoot	number of bootstrap samples if bootstrap = TRUE

Details

four methods used to compute p value of global F test

- truncated: asymptotic inference, p-value is obtained by truncating the infinite summation of eigenvalues into the first K terms, where the first K terms explain more than 99.99% of the variance.
- satterthwaite: asymptotic inference, p-value is computed using Satterthwaite's approximation method of mixtures of chi-square.
- permutation: resampling technique; Wasserstein SSR is used as the F statistic.
- bootstrap: resampling technique; Wasserstein SSR is used as the F statistic.

Value

a list containing the following fields:

wasserstein.F_stat	the Wasserstein F statistic value in Satterthwaite method .
chisq_df	the degree of freedom of the null chi-square distribution.
summary_df	a dataframe containing the following columns:

- method: methods used to compute p value, see details
- statistic: the test statistics
- critical_value: critical value
- p_value: p value of global F test

Examples

```
data(strokeCTdensity)
predictor = strokeCTdensity$predictors
dSup = strokeCTdensity$densitySupport
densityCurves = strokeCTdensity$densityCurve

res = wass_regress(rightside_formula = ~., Xfit_df = predictor,
 Ytype = 'density', Ymat = densityCurves, Sup = dSup)
globalF_res = globalFtest(res, alpha = 0.05, permutation = TRUE, numPermu = 200)
```

`partialFtest` *partial F test for Wasserstein regression*

Description

partial F test for Wasserstein regression

Usage

```
partialFtest(reduced_res, full_res, alpha = 0.05)
```

Arguments

`reduced_res`	a reduced model list returned by the `wass_regress` function
`full_res`	a full model list returned by the `wass_regress` function
`alpha`	type one error rate

Details

two methods used to compute p value using asymptotic distribution of F statistic

- truncated: asymptotic inference, p-value is obtained by truncating the infinite summation of eigenvalues into the first K terms, where the first K terms explain more than 99.99% of the variance.
- satterthwaite: asymptotic inference, p-value is computed using Satterthwaite approximation method of mixtures of chi-square.

Value

a dataframe containing the following columns:

`method`	methods used to compute p value, see details
`statistic`	the test statistics
`critical_value`	critical value
`p_value`	p value of global F test

Examples

```
data(strokeCTdensity)
predictor = strokeCTdensity$predictors
dSup = strokeCTdensity$densitySupport
densityCurves = strokeCTdensity$densityCurve

full_res <- wass_regress(rightside_formula = ~., Xfit_df = predictor,
 Ymat = densityCurves, Ytype = 'density', Sup = dSup)
reduced_res <- wass_regress(~ log_b_vol + b_shapInd + midline_shift + B_TimeCT, Xfit_df = predictor,
 Ymat = densityCurves, Ytype = 'density', Sup = dSup)
partialFtable = partialFtest(reduced_res, full_res, alpha = 0.05)
```

print.summary.WRI *print the summary of WRI object*

Description

print the summary of WRI object

Usage

```
## S3 method for class 'summary.WRI'
print(x, ...)
```

Arguments

x	a 'summary.WRI' object
...	further arguments passed to or from other methods.

quan2den_qd *convert density function to quantile and quantile density function*

Description

convert density function to quantile and quantile density function

Usage

```
quan2den_qd(quantileCurves, t_vec)
```

Arguments

quantileCurves	n-by-m matrix of quantile curves
t_vec	length m vector contains the common support grid of the quantile curves

`simulate_quantile_curves` *Simulate quantile curves*

Description

This function simulates quantile curves used as a toy example

Usage

```
simulate_quantile_curves(x1, alpha, beta, t_vec)
```

Arguments

`x1`	n-by-1 predictor vector
`alpha`	parameter in location transformation
`beta`	parameter in variance transformation
`t_vec`	a length m vector - common grid for all quantile functions

Value

quan_obs n-by-m matrix of quantile functions

References

Wasserstein F-tests and confidence bands for the Frechet regression of density response curves, Alexander Petersen, Xi Liu and Afshin A. Divani, 2019

Examples

```
alpha = 2
beta = 1
n = 100
x1 = runif(n)
t_vec = unique(c(seq(0, 0.05, 0.001), seq(0.05, 0.95, 0.05), seq(0.95, 1, 0.001)))
quan_obs = simulate_quantile_curves(x1, alpha, beta, t_vec)
```

strokeCTdensity *Stroke data: clinical, radiological scalar variables and density curves of the hematoma of 393 stroke patients*

Description

Stroke data: clinical, radiological scalar variables and density curves of the hematoma of 393 stroke patients

Format

a list of the following three fields:

densityCurve: 393-by-101 head CT hematoma densities as distributional response

densitySupport: length 101 common support vector

predictors: 393-by-9 matrix containing 9 scalar predictors

References

Wasserstein F-tests and confidence bands for the Frechet regression of density response curves, Alexander Petersen, Xi Liu and Afshin A. Divani, 2019

summary.WRI *Summary Function of Wasserstein Regression Model*

Description

Summary Function of Wasserstein Regression Model

Usage

```
## S3 method for class 'WRI'
summary(object, ...)
```

Arguments

object	an object returned by the wass_regress function
...	further arguments passed to or from other methods.

Value

a list containing the following fields:

`call` function call of the Wasserstein regression

`r.square` Wasserstein R^2, the Wasserstein coefficient of determination

`global_wasserstein_F_stat`
Wasserstein global F test statistic from the Satterthwaite method

`global_F_pvalue`
p value of global F test

`global_wasserstein_F_df`
degrees of freedom of satterthwaite approximated sampling distribution used in global F test

`partial_F_table`
Partial F test for individual effects

Examples

```
data(strokeCTdensity)
predictor = strokeCTdensity$predictors
dSup = strokeCTdensity$densitySupport
densityCurves = strokeCTdensity$densityCurve

res <- wass_regress(rightside_formula = ~., Xfit_df = predictor,
Ymat = densityCurves, Ytype = 'density', Sup = dSup)
summary(res)
```

---

wass_R2 *Compute Wasserstein Coefficient of Determination*

Description

Compute Wasserstein Coefficient of Determination

Usage

```
wass_R2(wass_regress_res)
```

Arguments

`wass_regress_res`
an object returned by the `wass_regress` function

Value

Wasserstein R^2, the Wasserstein coefficient of determination

References

Frechet regression for random objects with Euclidean predictors, Alexander Petersen and Hans-Georg Müller, 2019

Examples

```
data(strokeCTdensity)
predictor = strokeCTdensity$predictors
dSup = strokeCTdensity$densitySupport
densityCurves = strokeCTdensity$densityCurve

res = wass_regress(rightside_formula = ~., Xfit_df = predictor,
Ymat = densityCurves, Ytype = 'density', Sup = dSup)
wass_r2 = wass_R2(res)
```

`wass_regress` *Perform Frechet Regression with the Wasserstein Distance*

Description

Perform Frechet Regression with the Wasserstein Distance

Usage

```
wass_regress(rightside_formula, Xfit_df, Ytype, Ymat, Sup = NULL)
```

Arguments

`rightside_formula`	a right-side formula
`Xfit_df`	n-by-p matrix (or dataframe) of predictor values for fitting (do not include a column for the intercept)
`Ytype`	'quantile' or 'density'
`Ymat`	one of the following matrices: • if Ytype = 'quantile' Ymat is an n-by-m matrix of the observed quantile functions. Ymat[i, :] is a 1-by-m vector of quantile function values on grid `Sup`. • if Ytype = 'density' Ymat is an n-by-m matrix of the observed density functions. Ymat[i, :] is a 1-by-m vector of density function values on grid `Sup`.
`Sup`	one of the following vectors: • if Ytype = 'quantile' Sup is a length m vector - common grid for all quantile functions in Ymat (default: seq(0, 1, length.out = ncol(Ymat))). • if Ytype = 'density' Sup is a length m vector - common grid for all density functions in Ymat (default: seq(0, 1, length.out = ncol(Ymat))).

Value

a list containing the following objects:

call	function call
rformula	rightside_formula
predictor_names	names of predictors as the colnames given in the xfit matrix or dataframe.
Qfit	n-by-m matrix of fitted quantile functions.
xfit	design matrix in quantile fitting.
Xfit_df	n-by-p matrix (or dataframe) of predictor values for fitting
Yobs	a list containing the following matrices: • Qobs: n-by-m matrix of the observed quantile functions. • qobs: n-by-m matrix of the observed quantile density functions. • qobs_prime: n-by-m matrix of the first derivative of the observed quantile density functions. • fobs: n-by-m matrix of the observed density functions.
t_vec	a length m vector - common grid for all quantile functions in Qobs.

References

Wasserstein F-tests and confidence bands for the Frechet regression of density response curves, Alexander Petersen, Xi Liu and Afshin A. Divani, 2019

Examples

```
data(strokeCTdensity)
predictor = strokeCTdensity$predictors
dSup = strokeCTdensity$densitySupport
densityCurves = strokeCTdensity$densityCurve

res1 = wass_regress(rightside_formula = ~., Xfit_df = predictor,
 Ytype = 'density', Ymat = densityCurves, Sup = dSup)
res2 = wass_regress(rightside_formula = ~ log_b_vol * weight, Xfit_df = predictor,
 Ytype = 'density', Ymat = densityCurves, Sup = dSup)
```

Index